compression distance in meters? (b) How much vertical force does the jumper apply to the pogo stick? (c) How much vertical force does the pogo stick exert on the jumper?

3. [I] A helical spring 20-cm long extends to a length of 25 cm when it supports a load of 50 N. Determine the spring constant. [*Hint: Hooke's Law governs the extension of springs.*]

4. [I] When an object weighing 200 N is hung from a vertical spring, the spring stretches 10.0 cm. Calculate the spring's elastic constant.

5. [I] A steel spring is suspended vertically from its upper end, and a monkey weighing 10.0 N grabs hold of its bottom end and hangs motionlessly from it. If the elastic constant of the spring is 500 N/m, by how much will the monkey stretch it?

6. [I] Given a spring with an elastic constant of 500 N/m, how much will it contract when pushed on by an axial force of 10 N?

7. [I] An ordinary helical spring having an elastic constant of 2.00 N/cm is stretched 10.0 cm and held in that configuration. How much work was thereby done on the spring?

SOLUTION: The work done equals the energy stored, and that's $\Delta PE_e = \frac{1}{2}ks^2 = \frac{1}{2}(2.00 \times 10^2\ N/m)(0.100\ m)^2 = 1.00\ J$.

8. [I] A long metal wire hangs from the roof truss in a factory building. A 1000-kg machine is attached to it so that the load is suspended above the floor. If the wire stretches 0.500 cm, what is its elastic constant?

9. [I] When a bowstring is pulled back in preparation for shooting an arrow, the system behaves in a Hookean fashion. Suppose the string is drawn 0.700 m and held with a force of 450 N, what is the elastic constant of the bow?

10. [I] How much energy is stored in a helical spring with an elastic constant of 50 N/m when compressed 0.05 m?

11. [II] THIS PROBLEM EXPLORES THE STATIC RESPONSE OF A LOADED SPRING. A helical spring with an elastic constant of 1000 N/m is suspended vertically from its top. A mass of 7.99 kg is attached to the spring's bottom and the system comes to equilibrium. (a) What is the value of the force applied to the bottom of the spring? (b) Does Hooke's Law apply? (c) By how much is the spring stretched by the load? (d) How much work was done on the spring in the process of stretching it?

12. [II] THIS PROBLEM DEALS WITH THE BEHAVIOR OF AN IDEAL SPRING. A jack-in-the-box is built around a spring. It takes 4.4 N to hold the lid down on his little head after he's compressed 15 cm from his sprung equilibrium length. (a) What is the value of the force applied by the jack on the lid? (b) By how much in meters is the spring compressed while in the box? (c) What's the value of Jack's spring constant? (d) How much work was done on Jack in the process of getting him into the box?

13. [II] A helical spring is 55-cm long when a load of 100 N is hung from it and 57-cm long when the load is 110 N. Find its spring constant.

14. [II] Two identical helical springs are attached to one another, end-to-end, making one long spring. If $k = 500$ N/m is the elastic constant of each separate (essentially weightless) spring, what is the constant for the combination?

15. [II] A Hookean spring is suspended vertically, and a mass of 2.00 kg is hung from its end. The spring then stretches 10.0 cm. How much more will it elongate if an additional 0.50 kg is attached to the first mass?

16. [II] Two identical wires each of length L_0 are attached to one another producing a single long wire. If $k = 500$ N/m is the elastic constant of each separate (essentially weightless) wire, what is the constant for the combination?

SOLUTION: Suppose the load on the combined wire is F. Each wire is then under a tension of F, and each stretches an amount s as if it were supporting the load all by itself. Each length of wire elongates by $s = F/k$, giving a total elongation of 2s. Hence, for the composite wire $k' = F/2s = k/2 = 250$ N/m.

17. [II] Imagine a horizontal spring with an elastic constant of 25.0 N/cm, held in place just above an air table. If a 2.00-kg mass traveling at 1.50 m/s slams axially into the end of the spring, by how much will it be compressed in bringing the mass to rest?

18. [III] During the filming of a movie a 100.0-kg stunt man steps off the roof of a building and free-falls. He is attached to a safety line 50.0-m long that has an elastic constant of 1000 N/cm. What will be the maximum stretch of the line at the instant he comes to rest, assuming it remains Hookean?

SECTION 10.2: STRESS & STRAIN

SECTION 10.3: STRENGTH

19. [I] A vertical iron rod, having a known length and diameter, is to support a given load. If the rod is replaced by a new rod twice as long, and all else is kept constant, compare the stresses on both rods.

20. [I] Hanging vertically, a steel wire of a length L and radius R is to support a given load. If its cross-sectional area is halved and all else is the same, what will happen to the stress on the wire?

21. [I] An aluminum bar of known length and diameter is supported vertically, and a given load is attached to its lower end. If this bar is replaced by one having half the previous diameter, and everything else is unchanged, compare the stresses on the two bars.

22. [I] Standing vertically a copper cylinder of a length L and diameter D supports a compressive load placed on its top. An identical cylinder having half the diameter supports half the original load. If all else is the same, compare the stresses on the two cylinders?

23. [I] When a mass of 4.00 kg is suspended from the end of a narrow metal rod having a radius of 0.707 mm, the rod stretches by 0.060 cm. Determine the normal stress in the rod.

SOLUTION: $\sigma = F/A = mg/\pi R^2 = (4.00\ kg)(9.81\ m/s^2)/(3.142)(0.707 \times 10^{-3}\ m)^2 = 2.50 \times 10^7$ Pa.

24. [I] When a mass of 4.00 kg is suspended from the end of a narrow metal rod 60.0-cm long having a diameter of 0.707 mm, the rod stretches by 0.060 cm. Determine the normal strain on the rod.

25. [I] An axial force of 200 N is applied to a rod with a cross-sectional area of 10^{-4} m^2. What is the normal stress within the rod?

26. [I] A man leans down on a cane with a vertical axial force of 100 N. The narrowest part of the cane has a cross-sectional area of 1.0 cm^2. What is the maximum compressive stress in the cane?

27. [I] A 10-m-long wire strung between two trees has a "Curb Your Dog" sign weighing 10 N hung from it. If the wire stretches 1.5 cm, what strain is it under?

a maximum stress of 150 MPa without rupturing and has a Young's Modulus (in compression) of 10 GPa. How much elastic energy can be stored in these two leg supports (as a result of jumping in a gravity field) without having to go back to the shop for repair? If the robot has a mass of 70 kg, how high a drop can it sustain on Earth?

SECTION 10.5: SIMPLE HARMONIC MOTION

65. [I] What is the period of an old fashioned phonograph record that turns through $33\frac{1}{3}$ rotations per minute?

66. [I] The respiratory system of a medium-sized dog resonates at roughly 5 Hz so that it can pant (in order to cool off) very efficiently at that frequency. How many breaths will the dog be taking in a minute? For comparison, the dog would ordinarily breathe at around 30 breaths per minute.

67. [I] An antique phonograph record is turning uniformly at 78 rpm while an ant sitting at rest on its rim is being viewed by a child whose eyes are in the plane of the record. Describe the ant's motion as seen by the child. What is the frequency of the ant? What is its angular frequency?

68. [I] A hovering fair-sized insect rises a little during the downstroke of its wings and essentially free-falls slightly during the upstroke. The end result is that the creature oscillates in midair. If it typically falls about 0.20 mm per cycle, what is the wingbeat period and frequency? Compare that to the $\approx$10 Hz oscillation of a butterfly, which cannot hover.

69. [I] A point at the end of a spoon whose handle is clenched between someone's teeth vibrates in SHM at 50 Hz with an amplitude of 0.50 cm. Determine its acceleration at the extremes of each swing.

70. [I] A light hacksaw blade is clamped horizontally, and a 5.0-g wad of clay is stuck to its free end, 20 cm from the clamp. The clay vibrates with an amplitude of 2.0 cm at 10 Hz. (a) Find the speed of the clay as it passes through the equilibrium position. (b) Determine its acceleration at maximum displacement and compare that with g.

71. [I] A small mass is vibrating in SHM along a straight line. Its acceleration is 0.40 m/s^2 when at a point 20 cm from the zero of equilibrium. Determine its period of oscillation.

72. [I] A point on the very end of one-half of a tuning fork vibrates approximately as if it were in SHM with an amplitude of 0.50 mm. If the point returns to its equilibrium position with a speed of 1.57 m/s, find the frequency of vibration.

73. [I] A body oscillates in SHM according to the equation

$$x = 5.0\cos(0.40t + 0.10)$$

where each term is in SI units. What is (a) the amplitude, (b) the frequency, and (c) the initial phase at $t = 0$? (d) What is the displacement at $t = 2.0$ s?

74. [I] A body oscillates in SHM according to the equation

$$x = 8.0\cos(1.2t + 0.4)$$

where each term is in SI units. What is the period?

75. [I] THIS PROBLEM USES FIGURE 10.23 TO PROVE THAT IN SHM

$$T = 2\pi A/V_{max}$$

(a) What is the speed at which the little object travels around the circle? (b) What is the distance once around the circle? (c) Write an expression for the time it takes for the object to go once around the circle.

76. [II] THIS PROBLEM EXPLORES SHM FOR AN IDEAL MASS-SPRING SYSTEM. A 0.802-kg bob hangs in equilibrium from a lightweight spring. It is pulled downward by a positive amount and released. It subsequently oscillates in SHM according to the equation

$$y = 0.760\cos(5.50t)$$

where each term is in SI units. (a) Write a general expression for the displacement of an oscillator whose bob is at $+A$ at $t = 0$. What is (b) the amplitude of the oscillation, (c) the angular frequency, (d) the linear frequency, and (e) the period? (f) What is the bob's displacement at $t = (\frac{1}{2}\pi/5.50)$ s? (g) How fast is it moving at that time? (h) How fast is it moving at $t = (\pi/5.50)$ s? (i) What's the value of its acceleration at that moment?

77. [II] Suppose the general equation $x = A\cos(\omega t + \epsilon)$ is to be applied to the mass in Fig. 10.26, where the cycle is to begin at $t = 0$ with m released with zero speed at -3 m. What must ϵ equal?

SOLUTION: At $t = 0$, $x = A\cos(0 + \epsilon) = A\cos\epsilon = -3$ m. Whereas $v_x = -A\omega\sin(\omega t + \epsilon) = -A\omega\sin(0 + \epsilon) = -A\omega\sin\epsilon = 0$ and so $\sin\epsilon = 0$ and $\epsilon = 0$ or π. To see which is the case, recall that $A\cos\epsilon = -3$ m and that's < 0. Since A is always > 0 this means $\cos\epsilon < 0$. But $\cos\epsilon < 1$ so $\epsilon = \pi$.

78. [II] Suppose the general equation $x = A\cos(\omega t + \epsilon)$ is to be applied to the mass in Fig. 10.25, where the timing clock is started arbitrarily and it is found that at $t = \frac{1}{4}T$ the mass is at $x = +\frac{1}{2}A$. Find ϵ.

79. [II] A uniform block with sides of length a, b, and c floats partially submerged in water. It is pushed down a little and let loose to oscillate. Given that the vertical edge has length b and the density of the block is ρ, show that the motion is SHM and determine its period. Check the units of your result.

80. [II] A 5.0-kg block of wood is floating in water. It is found that a downward thrust of 10.0 N submerges the block an additional 10 cm, at which point it is let loose to oscillate up and down. Determine the frequency of the oscillation. Assume SHM.

81. [II] Show that a value of $\epsilon = +\frac{3}{2}\pi$ is equivalent to a value of $\epsilon = -\frac{1}{2}\pi$ in the general equation $x = A\cos(\omega t + \epsilon)$. Draw a sketch of the functions.

82. [II] A spot of light on a computer screen oscillates horizontally in SHM along a line 20-cm long at 50 Hz. The spot reaches the center of the line at $t = T/8$. Show that $v_x = -10\pi\sin(100\pi t + \frac{1}{4}\pi)$ is the expression for the speed of the spot as a function of time.

83. [II] A particle is oscillating with SHM along the z-axis with an amplitude of 0.50 m and a frequency of 0.20 Hz. If it is at $z = +0.50$ m at $t = 0$, where will it be at $t = 5.00$ s, $t = 2.50$ s, and $t = 1.25$ s?

84. [III] A vibration platform oscillates up and down with an amplitude of 10 cm at a controlled variable frequency. Suppose a small rock of mass m is placed on the platform. At what frequency will the rock just begin to leave the surface so that it starts to clatter?

85. [III] Use Fig. P85 to prove that the reciprocating motion of the piston is oscillatory but not SHM.

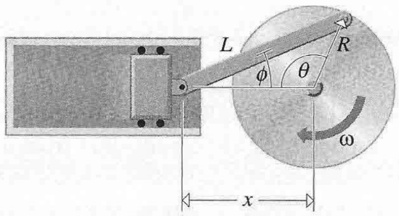

Figure P85

SECTION 10.6: ELASTIC RESTORING FORCE

86. [I] A 1.0-N bird descends onto a branch that bends and goes into SHM with a period of 0.50 s. Determine the effective elastic force constant for the branch.

87. [I] A bug having a mass of 0.20 g falls into a spider's web, setting it into vibration with a dominant frequency of 18 Hz. Find the corresponding elastic spring constant.

88. [I] A 1.0-N weight is attached to a vertical spring and it's then set oscillating up and down. Later three additional 1.0-N weights are added on to the bottom of the spring and it's again set oscillating vertically. Compare the periods of the two oscillations.

89. [I] THIS PROBLEM IS ABOUT A SPRING VIBRATING IN SHM. A hanging scale in a vegetable market oscillates at 3.0 Hz after a 1.20-kg bag of potatoes is dropped on it. If the pan of the scale has a mass of 0.50 kg, (a) what is the total mass supported by the spring? (b) What is the natural frequency of the system? (c) Determine the elastic constant of the spring.

90. [I] THIS PROBLEM IS ABOUT AN ELASTIC SYSTEM VIBRATING IN SHM. A 85.0-kg man tied to a 25.0-m bungee cord steps of a bridge. He just misses the water, at which point the cord is stretched to 45.0 m in length. We want to find out how much time will elapse from when he leaves the bridge to when he nearly returns on the first bounce. (a) What's the maximum stretch of the cord? (b) How much *gravitational*-PE did he have before he left the bridge? (c) What was the maximum amount of elastic energy stored in the cord? (d) Determine the cord's elastic constant. (e) Compute the period of the oscillation.

91. [I] THIS PROBLEM APPLIES ENERGY CONSIDERATIONS TO SHM. A mass m on a horizontal frictionless table is attached to the end of a spring with an elastic constant of k. The mass has a scalar velocity of v_1 when it's at x_1. (a) Write an expression for the total energy of the oscillator. (b) Write an expression for the maximum potential energy of the oscillator in terms of A, the amplitude of the oscillation. (c) Write an expression for A in terms of m, k, v_1, and x_1.

92. [I] A 5.0-kg mass resting on a frictionless airtable is attached to a spring with an elastic constant of 50 N/m. If this mass is displaced 10 cm (compressing the spring) and is then released, find its maximum speed.

93. [I] Two kilograms of potatoes are put on a scale that is displaced 2.50 cm as a result. What is the elastic spring constant? If the scale is pushed down a little and allowed to oscillate, what will be the frequency of the motion?

94. [I] A 250-g mass is attached to a light helical spring with an elastic force constant of 1000 N/m. It is then set into SHM with an amplitude of 20 cm. Determine the total energy of the system.

95. [I] Consider the frictionless cart in Fig. 10.27. If the elastic spring constants are k_1 and k_2, respectively, determine the frequency of vibration in terms of these quantities.

96. [I] What is the speed of the mass in Problem 92 as it reaches a point 5.0 cm from its undisplaced position?

97. [II] A light 3.0-m-long helical spring hangs vertically from a tall stand. A mass of 1.00 kg is then suspended from the bottom of the spring, which lengthens an additional 50 cm before coming to a new equilibrium configuration well within its elastic limit. The bob is now pushed up 10 cm and released. Write an equation describing the displacement y as a function of time.

98. [II] A 200-g mass hung on the bottom of a light helical spring stretches it 10 cm. This mass is then replaced with a new 500-g bob that is set into SHM. Compute its period.

99. [II] An object is hung from a light vertical helical spring that subsequently stretches 2.0 cm. The body is then displaced and set into SHM. Compute the frequency at which it oscillates.

100. [II] Derive an expression for the period of oscillation of the frictionless system shown in Fig. P100. Remember that the displacement of the mass m is the sum of the displacements of the two springs.

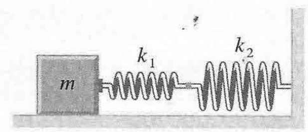

Figure P100

101. [II] An essentially weightless helical spring hangs vertically. When a mass m is suspended from it, it elongates an amount ΔL. When let loose, it oscillates with a measured period of T. Show how you might use this arrangement to determine g.

102. [II] THIS PROBLEM APPLIES ENERGY CONSIDERATIONS TO SHM. A 0.70-kg mass at rest on a horizontal frictionless table is attached to the end of an ideal spring having an elastic constant of 90 N/m. A second 0.30-kg mass is launched toward the first at a speed of 18.0 m/s. The two masses collide and stick together. We wish to write an expression describing the resulting motion. Assuming momentum is conserved in the collision, (a) at what speed does the combined mass move immediately after impact? (b) How much KE is imparted to the composite oscillator via the collision? (c) What is the amplitude of the resulting oscillation? (d) Determine the angular frequency of the system. (e) Keeping in mind that $x = 0$ at $t = 0$, write an expression for the oscillatory displacement.

103. [II] THIS PROBLEM DEALS WITH THE ENERGY OF A MASS-SPRING SYSTEM IN SHM. Considering the system in Problem 76 (a) What is the value of the spring constant? (b) What is the system's maximum potential energy? (c) Assuming it to be lossless, what is its total energy at any moment? (d) Determine the bob's displacement 3.0 s after release. (e) What is its potential energy at that time? (f) What is the KE of the bob at that time? (g) What is the speed of the bob at that moment?

104. [II] A 5.0-g bullet is fired horizontally into a 0.50-kg block of wood resting on a frictionless table. The block, which is attached to a horizontal spring, retains the bullet and moves forward, compressing the spring. The block-spring system goes into SHM with a frequency of 9.0 Hz and an amplitude of 15 cm. Determine the initial speed of the bullet.

105. [II] Using energy considerations, derive an expression for the period of oscillation of a mass m. It is fixed to a horizontal spring having an elastic constant k and is free to move on a frictionless table.

SOLUTION: For SHM the maximum KE equals the maximum PE: $PE_{max} = \frac{1}{2}kA^2 = KE_{max} = \frac{1}{2}mv_{max}^2$ and so $A/v_{max} = \sqrt{m/k}$. Using the results of Problem 75 namely, $T = 2\pi A/v_{max} = 2\pi\sqrt{m/k}$

SECTION 10.7: THE PENDULUM

106. [I] What happens to the period of a simple pendulum if we increase its length so that it's four times longer?

107. [I] What happens to the frequency of a simple pendulum if we increase its length so that it's four times longer?

108. [I] What happens to the period of a simple pendulum if we increase its mass so that it's four times greater?

109. [I] Find the frequency of a simple pendulum of length 10.0 m.

110. [I] A small lead ball is attached to a light string so that the length from the center of the ball to the point of suspension is 1.00 m. Determine the natural period of the ball swinging through a small displacement in a vertical plane.

111. [I] How long must a simple pendulum be if it is to have a period of 10.0 s? [*Hint: The period of a simple pendulum is proportional to the square root of the length.*]

112. [I] While standing on a distant planet, it is found that a 1.00 m long pendulum has a period of 10.00 s. What is the acceleration due to gravity on that planet?

113. [I] How long must a simple pendulum be if it is to have a period of 10.00 s on Mars where the acceleration due to gravity is 39% of its value on Earth?

114. [I] One might guess that the period of a pendulum is proportional to the mass m, the length L, and the acceleration due to gravity g. Consequently, assume that

$$T = Cm^a L^b g^c$$

where C is a unitless constant. Now solve for a, b, and c using the fact that the dimensions on both sides of the equation must be the same. Of course, if this calculation produces nonsense, C wasn't unitless. Although you cannot get it from this analysis, what is the correct value of C?

115. [I] What happens to the frequency of a simple pendulum if we move it to the Moon's surface where the acceleration due to gravity is 1/6 that at the Earth's surface?

116. [II] THIS PROBLEM WILL HELP US TO FURTHER UNDERSTAND THE DEPENDENCE OF A PENDULUM'S MOTION ON GRAVITY. We need an equation for the period of a pendulum at any distance r from the center of the Earth. (a) What does the period of a pendulum depend on? (b) How does g vary with r? (c) Write an equation for T as a function of r.

117. [II] THIS PROBLEM EXAMINES THE DEPENDENCE OF A PENDULUM'S MOTION ON GRAVITY. A pendulum at the surface of the Earth has a period T_s. It is taken to an altitude h where its period is then T_h. We want to prove that

$$h = r_{\oplus}\left(\frac{T_h}{T_s} - 1\right)$$

(a) With the previous problem in mind, write an expression for T_h in terms of r. (b) Express r in terms of h and the Earth's radius $r_{\oplus}$. (c) Write an expression for T_s in terms of $r_{\oplus}$. (d) Write an equation for T_h/T_s. (e) Now derive the above formula for h.

118. [II] THIS PROBLEM TREATS THE MOTION OF A PENDULUM UNDER VARIOUS CIRCUMSTANCES. A pendulum of length L_1 has a period T_1, at a location where the acceleration due to gravity is $\frac{1}{2}g$. We want to find out how long a second pendulum should be if its period is to be $4T_1$ at a location where the acceleration due to gravity is g? (a) Write an expression for T_1 in terms of L_1 and g. (b) Write an expression for T_2 in terms of L_2 and g. (c) Write an expression for T_2 in terms of T_1. (d) Write an expression for L_2 in terms of L_1.

119. [II] THIS PROBLEM TREATS THE MOTION OF A PENDULUM FROM THE ENERGY PERSPECTIVE. The mass of the bob in a simple pendulum has a maximum speed of 3.0 m/s and we wish to compute the height (h) to which it rises at the end of each swing. (a) Write an expression for the maximum KE of the bob. (b) Write an expression for the maximum PE_G of the bob. (c) What can you say about these last two expressions? (d) Now create an expression for the maximum height. (e) Compute the height at each end of its swing.

120. [II] Consider a simple pendulum of length L. Show that it gains an amount of potential energy

$$\Delta PE_G \approx \tfrac{1}{2}mgL\theta^2$$

when the bob is raised through an angle θ. [*Hint: You may need to know that* $1 - \cos\theta = 2\sin^2\frac{1}{2}\theta$.]

121. [II] If the period of a simple pendulum is T, what will its new period be if its length is increased by 50%?

122. [II] Extending the ideas of Problem 120, show that the pendulum's maximum angular speed is given by $\omega_{max} = \theta_{max}\sqrt{g/L}$.

123. [II] A pendulum consists of a 2.00-cm-diameter lead sphere attached to a frictionless pivot via a long very light cable. It is to swing from its maximum displacement on one side to its maximum displacement on the other side in 1.00 s. If it's to do this at a place where the acceleration due to gravity is 9.80 m/s², how long must the cable be?

124. [II] THIS PROBLEM EXPLORES THE DEPENDENCE OF A PENDULUM'S PERIOD ON ITS ACCELERATION. We want to derive an expression for the period of a pendulum swinging vertically in an elevator that is itself accelerating vertically at a rate a. (a) Imagine the elevator to be accelerating uniformly upward. Would you seem to weigh more or less in such an environment? (b) If we imagine a to be produced by a change in the gravitational field acting on a stationary elevator should we increase or decrease g? (c) Now suppose the elevator to be accelerating uniformly downward. Would you seem to weigh more or less? (d) If we imagine a to be pro-