

tions. Then colinear vectors will properly add (or subtract) as scalars. If the initial velocity vectors are not colinear, their separate component momenta in, say, the x - and y -directions will be conserved individually. Be very careful with the signs in momentum problems.

5. We are restricted to the two extreme classes of collision problems, and that's helpful to remember. If the colliding bodies stick together, the collision involves only Conservation of Momentum and there will be one equation and only one unknown. When the two bodies move off together, there will be one final velocity for both. By contrast, in one dimension, if the collision is elastic, we can also

use Conservation of Energy, and there will be two equations and there can be two unknowns. The final velocities of the bodies will be different.

6. Proceed cautiously when confronted with a collision problem where it's not explicitly stated that mechanical energy is conserved. A bullet that squashes against a steel plate or imbeds in a phone book or passes right through a wooden board will "lose" an appreciable amount of energy. Nonetheless, provided no external forces are acting on the bullet-board system, momentum will be conserved. Watch out for situations where any of the objects taking part suffer deformation or generate "heat" via friction.

Problems + Coordinated Problems + Progressive Problems + Solutions

STUDY GUIDE

1. Coordinated Problems: The three problems within each magenta-colored grouping are solvable in similar ways. Note that the first of these always has a hint; moreover, its solution is provided in the back of the book. *Work out each of these sets; they'll strengthen technique and build confidence.* **2. Progressive Problems:** The problems introduced in blue unfold step-by-step carrying along the analysis in a more suggestive way than is customary. *Work out all of these; they'll guide you through the analytic process and help develop problem-solving skills.* **3. Worked-Out Solutions:** Studying worked-out solutions is an important part of learning how to solve problems. Accordingly, additional **solutions** to a number of model problems are given below. *Make sure you understand each of them before you go on to the next problem.* **4.** Also provided in the back of the book are the **Answers** to all odd-numbered problems, as well as worked-out **solutions** to those with boldface numbers. Problem numbers in italic indicate that a solution appears in the Student Solutions Manual.

SECTION 7.1: IMPULSE AND MOMENTUM CHANGE

SECTION 7.2: VARYING FORCE

1. [I] A 400-kg L75-domestic robot is flying due north down the emergency shaft of a space freighter at 12.0 m/s. Determine its momentum.

2. [I] If the magnitude of the momentum of a 47-g golf ball is 2.8 kg·m/s, what's its speed?

3. [I] From how high (in meters) must a car fall if it's to have the same momentum it would have driving at 60 mi/h (i.e., 60×0.4470 m/s)?

4. [I] What is the linear momentum of the Earth ($m_{\oplus} = 5.975 \times 10^{27}$ g) as it moves through space if its orbital speed is about 6.66×10^4 mi/h (i.e., $6.66 \times 10^4 \times 0.4470$ m/s)?

5. [I] A 1.0-kg wad of clay is slammed straight into a wall at a speed of 10 m/s. If the clay sticks in place, what was the impulse that acted on it via the wall? [*Hint: Take the direction of motion as positive, and compute the change in momentum of the clay.*]

6. [I] What is the impulse provided by the racket on a tennis ball (0.058 kg) served at 50 m/s?

7. [I] During a game a soccer ball having a mass of 0.425 kg is kicked from rest to 26 m/s in a collision with a player's foot lasting 8 ms. What is the impulse imparted to the ball?

8. [I] A dried pea fired from a plastic drinking straw has a mass of 0.50 g. If the force exerted on the pea is an average 0.070 lb (i.e., 0.070×4.448 N) over the 0.10-s flight through the straw, at what speed will it emerge?

9. [I] A 1.0-kg body initially traveling in the positive x -direction at 10 m/s is acted upon for 2.0 s by a force in that same direction of 20 N. It then experiences a force acting in the negative direction for 20 s equal to 2.0 N. Draw a force-time curve and determine the final momentum.

10. [I] A 10-kg asteroid is traveling through space at 2 m/s toward a spaceship. To avoid a collision, an astronaut, with the help of a backpack rocket, exerts a force of 20 N on the asteroid in the opposite direction for 5 s. (a) Find the asteroid's final speed. (b) How long should she have pushed in order to just stop the asteroid?

11. [I] If a 0.061-kg handball goes from rest to 20 m/s during a serve and the player feels an average force of 100 N, how long did the impact last?

SOLUTION: $F_{\text{av}} \Delta t = \Delta p$ and $\Delta p = (0.061 \text{ kg})(20 \text{ m/s}) = 1.22 \text{ kg}\cdot\text{m/s}$
hence $\Delta t = \Delta p / F_{\text{av}} = (1.22 \text{ kg}\cdot\text{m/s}) / (100 \text{ N}) = 1.2 \times 10^{-2} \text{ s}$.

12. [I] Figure P12 shows the force-time curves for two idealized blows delivered during a martial arts contest. Which of the two is likely to correspond to a boxer's punch as opposed to a karate chop? Compute an approximate value for the corresponding impulse in each case. Which is more likely to break bones, and why?

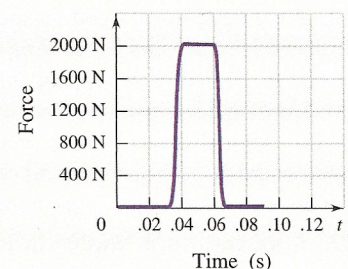
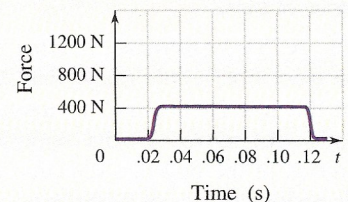


Figure P12

13. [I] Figure P13 is a force-time curve for a small frictionless cart. When, if ever, was its momentum constant?

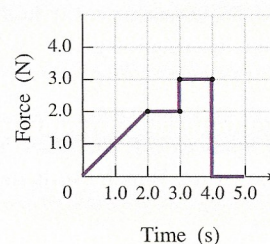


Figure P13

14. [I] Figure P13 is a force-time curve for a small frictionless cart. By how much did its momentum change during the interval from 0 to 2.0 s?

15. [III] **THIS PROBLEM EXAMINES THE SPEED OF AN OBJECT WHOSE FORCE-TIME CURVE IS GIVEN.** Figure P13

depicts the force applied to a frictionless cart, which was at rest at $t = 0$, and which has a mass of 0.50 kg. (a) What is the physical significance of the area under the curve? (b) Determine the area under the curve between $t = 0$ and $t = 4.0$ s. (c) How fast is the cart moving at $t = 4.0$ s?

16. [III] **THIS PROBLEM EXAMINES THE SPEED OF AN OBJECT WHOSE**

FORCE-TIME CURVE IS GIVEN. Figure P16 depicts the force applied to an ion engine (i.e., the thrust) mounted in a test vehicle whose total mass is 200.0 kg. (a) What is the physical significance of the area under the curve? (b) What is the significance of the fact that some of the area is below the axis? (c) Determine the area under the curve between $t = 0$ and $t = 9.54$ s. (d) If the craft is traveling through space at 10.0 m/s at $t = 0$, how fast will it be moving at $t = 9.54$ s?

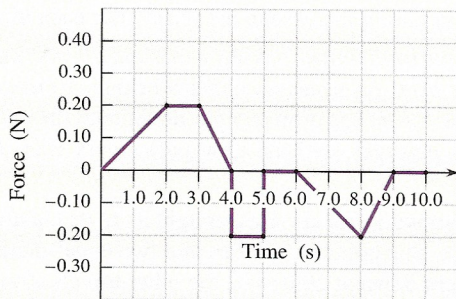


Figure P16

17. [II] A youngster having a mass of 50.0 kg steps off a 1.00-m high platform. If she keeps her legs fairly rigid and comes to rest in 10.0 ms, what is her momentum just as she hits the floor? What average force acts on her during the subsequent deceleration? [Hint: Use the fact that she falls with a constant acceleration to compute her impact speed.]

18. [III] A 20.0-kg ball of wet clay drops off a workman's scaffold and falls for 4.0 s before hitting the ground. If it comes to rest in 5.0 ms, what average force did the floor exert on the clay? Use momentum considerations to solve the problem.

19. [III] A 47-g golf ball is hit into the air at 60 m/s. It lands in sand at the same elevation and comes to rest in 10 ms. Ignoring air friction, what was the average force the sand exerted on the ball?

20. [II] A golf ball with a mass of 47.0 g can be blasted from rest to a speed of 70.0 m/s during the impact with a clubhead. Taking that impact to last only about 1.00 ms, (a) calculate the scalar value of the change in momentum of the ball. (b) What is the average force on the ball during the collision?

SOLUTION: Take the direction of motion of the club to be positive.

(a) $\Delta p = (0.047 \text{ kg})(70.0 \text{ m/s}) = 3.29 \text{ kg}\cdot\text{m/s}$. (b) $F_{\text{av}}\Delta t = \Delta p$; hence $F_{\text{av}} = \Delta p/\Delta t = (3.29 \text{ kg}\cdot\text{m/s})/(1.00 \times 10^{-3} \text{ s}) = 3.29 \times 10^3 \text{ N}$.

21. [II] A 1-kg hammer slams down on a nail at 5 m/s and bounces off at 1 m/s. If the impact lasts 1 ms, what average force is exerted on the nail?

22. [III] A person can just survive a full-body collision (either to the front, back, or side) at roughly 9 m/s (20 mi/h) with an impact time of approximately 10 ms. At greater speeds or shorter times, fatal brain damage will likely occur. Could someone survive a fall from 4.0 m landing flat on his back on soft soil so that he decelerates to rest through a distance of 10 cm (that's the total compression of body and soil)? If his mass is 70 kg, what's the impulse exerted on his body by the ground? Assume the deceleration is constant.

23. [II] The human heart pumps about 2 ounces of blood into the aorta on each stroke, which lasts roughly 0.1 s. During that time, the pulse of blood is accelerated from rest to about 50 cm/s. Compute the corresponding average propulsion force exerted on the blood by the heart.

24. [II] During a 40 mi/h (i.e., $40 \times 0.447 \text{ m/s}$) car crash, a 50-

kg passenger comes to rest in 100 ms. What is the area under the force-time curve for that person? Assuming a seat belt was worn, which would smooth out the curve so there were no devastating peaks, what average force would be experienced?

25. [II] A modern nuclear aircraft carrier weighs in at around 90 000 tons and is capable of traveling in excess of 30 knots (15.4 m/s). Suppose one of these giants plowed into a pier at 30 knots, coming to rest in 0.50 min. What average force would it exert in the process? (One ton, as used in the United States and Canada, equals 2 000 pounds.)

26. [III] A beam consisting of around 1.0×10^{15} electrons per second, each of mass $9.1 \times 10^{-31} \text{ kg}$, paints pictures on the face of a TV tube. Assuming that the electrons are absorbed by the screen and exert an average net force on it of $1.0 \times 10^{-7} \text{ N}$, what is their average speed at impact?

27. [II] An F-14A jet weighing 70 000 lb (i.e., $31.75 \times 10^3 \text{ kg}$) has a typical liftoff distance of 3000 ft (i.e., $3000 \times 0.3048 \text{ m}$) and a corresponding liftoff speed of 135 knots (69.5 m/s). Assuming its acceleration is constant, calculate the net average thrust of its engines in newtons.

28. [III] A typical tubular aluminum arrow 70-cm long has a mass of 25.0 g. The force exerted by a bow varies in a fairly complicated fashion from the initial draw force of, say, 175 N (39 lb) to zero, as the arrow leaves the string. Assume a full draw (70 cm) and a launch time of 16.0 ms. If the arrow when fired straight up reaches a maximum height of 143.3 m above the point of launch, then (a) What is the average force exerted on it by the bow? (b) What's the area under the force-time curve? (c) What impulse is applied to the shooter?

29. [III] The batter in a baseball game is ready to swing at a ball (of mass 0.149 kg) traveling toward him at 35.0 m/s horizontally. The bat strikes the ball, which flies upward at 45.0° at a speed of 39.0 m/s. Determine the impulse applied to the ball.

SECTION 7.3: ROCKETS

SECTION 7.4: CONSERVATION OF LINEAR MOMENTUM

30. [I] During the launch of a space rocket, fuel is consumed at a constant rate of 7.5 kg/s. If the thrust is to be 18 kN, determine the necessary exhaust speed.

31. [I] A rocket engine blasts out 1000 kg of exhaust gas at an average speed of 2 km/s every second. Calculate the resulting average thrust developed by the engine.

32. [I] Water squirting from a hose emerges with a speed of 10 m/s at a rate of 100 kg/s. Compute the average reaction force exerted on the hose. Have you ever seen firefighters working to control a large hose?

33. [I] **THIS PROBLEM DEALS WITH CONSERVATION OF LINEAR MOMENTUM.** Two gliders, one of mass 6.0 kg and the other 2.0 kg, are on a frictionless air track. They are locked together, thereby compressing a spring which is positioned between them. (a) Discuss the forces exerted by the spring on gliders. (b) When released they fly apart. What can be said about the impulse imparted to each glider? (c) What can be said about the momentum imparted to each glider? (d) The 6.0-kg glider travels at 3.0 m/s due east. What is the magnitude of its momentum? (e) What is the final velocity of the 2.0-kg glider?

34. [I] **THIS PROBLEM CONCERNS CONSERVATION OF LINEAR MOMENTUM.** A toy cannon having a mass of 4.0 kg sits on a frictionless lev-

el surface. It fires a 250-g projectile horizontally and immediately recoils westward at 1.5 m/s. (a) Find the change in momentum of the cannon. (b) Determine the change in momentum of the projectile. (c) What was the muzzle velocity of the projectile?

35. [I] A 50-kg person at the southernmost end of a 150-kg rowboat at rest in the water begins to walk to the northern end. If, at a given instant, her speed with respect to the water is 10 m/s, what is the velocity of the boat with respect to the water at that moment? [Hint: The initial momentum of the system (boat and person) is zero and momentum is conserved.]

36. [I] While constructing a space platform a 100-kg robot finds himself standing on a 30-m long 200-kg steel beam that is motionless with respect to the platform. Using his magnetic feet he walks along the beam traveling south at 2.00 m/s. What is the velocity of the beam with respect to the platform as the robot walks?

37. [I] The glider on an air track (Fig. Q1 in Chapter 4) is 30.0-cm long and has a mass of 0.500 kg. It floats frictionlessly on compressed air and is free to move along an east-west axis. A 100-g wind-up toy car is placed on the glider and it immediately rolls due east at 0.50 m/s with respect to the lab. Does the glider move and if so, how fast and in what direction?

38. [I] While floating in space a 100-kg robot throws a 0.800-kg wrench at 12.0 m/s toward his partner working on the spaceship. How fast will the robot move away from the ship?

39. [I] A 60-kg cosmonaut jumps from her 5000-kg spaceship in order to meet her copilot who is floating at rest a few hundred meters away. If she sails toward him at a speed of 10 m/s, what is the resulting motion of the ship?

40. [I] A kid holding a baseball bat stands at rest in a small boat. The total mass of the kid, bat, and boat is 250 kg. Someone on shore throws a 0.149-kg baseball at the boat. It flies straight down the length of the craft, reaching the batter at 20.0 m/s. He hits it with the bat and the ball sails back the way it came at 20.0 m/s. What's the speed of the boat immediately after the ball is hit?

41. [I] Two kids in a small boat, which is at rest on the water, play catch with a 1.00-kg lead ball. The total mass of the boat, ball, and kids is 250 kg. What is the speed of the boat just as the ball leaves the hand of the kid in the front at a speed of 5.00 m/s? In what direction does the boat move?

SOLUTION: Call the momentum of the ball \vec{p}_b and of the rest of the system \vec{p}_r . Since the initial momentum before the throw is zero, the net momentum after the throw is also zero. The ball travels toward the rear (take that to be the positive direction) and the boat moves forward. So $\vec{p}_b = -\vec{p}_r$ after the throw; $(1.00 \text{ kg})(5.00 \text{ m/s}) = -(249 \text{ kg})v_r$ and so $v_r = -2.01 \times 10^{-2} \text{ m/s}$.

42. [I] With Problem 41 in mind what is the speed of the boat just after the second kid catches the ball?

43. [II] Water with a density of $1.00 \times 10^3 \text{ kg/m}^3$ streams from a hose in a horizontal jet 2.4 cm in diameter at 22 m/s. It strikes a nearby vertical sponge-like absorbing wall, thereupon transferring all its kinetic energy to the wall so there is no splashing. What average force does the water exert on the wall?

44. [II] An ion engine is a rocket designed to produce small thrusts for very long periods of time. The SERT 2 engine develops 30 mN of thrust by expelling mercury ions at speeds of around 22 000 m/s (about 50 000 mi/h). How many kilograms of exhaust emerge per second?

45. [II] An ice skater (with a mass of 55.0 kg) throws a snowball while standing at rest. The ball has a mass of 200 g and moves straight out with a horizontal speed of 20.0 km/h. Neglecting friction, and assuming the skate blades are parallel to the direction of the throw, describe the skater's resulting motion in detail.

46. [II] A 90-kg astronaut floating out in space is carrying a 1.0-kg TV camera and a 10-kg battery pack. He's drifting toward his ship but, in order to get back faster, he hurls the camera out into space at 15 m/s and then throws the battery at 10 m/s in the same direction. What's the resulting increase in his speed after each throw?

47. [II] A railroad flatcar having a mass of 10 000 kg is coasting along at 20 m/s. As it passes under a bridge, 10 men (having an average mass of 90 kg) drop straight down onto the car. What is its speed as it emerges with its new passengers from beneath the bridge?

48. [II] A fireworks rocket rises straight up to its maximum altitude of 50.0 m. At that point it explodes into two equal-mass pieces; one heads straight down at 20.0 m/s, the other travels straight upward. What's the maximum altitude attained by the second upwardly-moving fragment?

49. [II] Two girls on roller skates, face-to-face, push on each other as hard as they can. Girl-1, whose mass is 20.0-kg, moves off west at 2.00 m/s. Write a general expression for the speed of the second body in such a situation and then calculate the specific speed of girl-2, whose mass is 40.0 kg.

50. [III] During a test a rocket is launched into a ballistic trajectory that ordinarily would have a range of 40.0 km. At the top of its parabolic arc across the sky it blows up, blasting into two equal-mass pieces. One drops straight down, while the other is hurled forward horizontally—where does it land?

51. [III] Two astronauts floating at rest with respect to their ship in space decide to play catch with a 0.500-kg asteroid. Neil (whose mass is 100 kg) heaves the asteroid at 20.0 m/s toward Sally (whose mass is 50.0 kg). She catches it and heaves it back at 20.0 m/s (with respect to the ship). Before Neil catches it a second time, how fast is each person moving, and in what direction?

SECTION 7.5: COLLISIONS

52. [I] Two identical cars of mass 2000 kg each drive toward one another, both traveling at 20.0 m/s. They collide head-on and smash together; what is the final speed of the wreckage?

53. [I] **THIS PROBLEM DEALS WITH CONSERVATION OF LINEAR MOMENTUM.** Two blocks on a frictionless air table slide directly toward each other. The first, traveling east has a momentum of 10.0 kg·m/s; the second traveling west has a momentum of 15.0 kg·m/s. They collide and more or less bounce off one another. (a) Taking east to be positive, what is the initial momentum of each block? (b) What is the total momentum of the two blocks before the collision? (c) What is the total momentum of the two blocks after the collision?

54. [I] **THIS PROBLEM IS ABOUT CONSERVATION OF LINEAR MOMENTUM.** Two carts, one of mass 9.0 kg and the other 3.0 kg, move frictionlessly on a horizontal surface. The 9.0-kg cart traveling north at 2.0 m/s crashes head-on into the 3.0-kg cart heading south and both slam to a stop. (a) Taking north to be positive, what is the initial momentum of the 9.0-kg cart? (b) What was the total momentum of the two carts after the collision? (c) What was the total momen-

tum of the two carts before the collision? (d) What was the momentum of the 3.0-kg cart prior to the collision? (e) Determine the speed of the 3.0-kg cart prior to the collision. (f) Was this an elastic collision?

55. [I] The glider on an air track has a mass of 1.00 kg and floats on compressed air, so it can move frictionlessly. Someone shoots a 20.0-g lump of clay at it. The clay strikes the glider, sticks to it, and both move away with a speed of 20.0 cm/s. What was the speed of the clay as it hit the glider?

SOLUTION: Momentum is conserved for the clay (C) and glider (G).

The collision is totally inelastic. Take the direction of motion of the clay as positive. Initially, $p_i = m_C v_{Ci}$; finally, $p_f = (m_C + m_G) v_f$ and since $p_i = p_f$, $m_C v_{Ci} = (m_C + m_G) v_f$. Putting the numbers in, $(20.0 \times 10^{-3} \text{ kg}) v_{Ci} = (1.02 \text{ kg})(20.0 \times 10^{-2} \text{ m/s})$ and $v_{Ci} = 10.2 \text{ m/s}$.

56. [I] Two wads of putty are propelled horizontally directly toward each other. Wad-1 has a mass of 5.00 kg and is traveling at 21.0 m/s south. Wad-2, which is 6.00 kg, is moving at 12.0 m/s north. They collide and stick together. What is the velocity of the joint mass of putty?

57. [I] A 90-kg signal relay floating in space is struck by a 1000-g meteoroid. The latter imbeds itself in the craft and the two sail away at 5.0 m/s. What was the initial speed of the meteoroid?

58. [I] A glass sphere of mass 100 g falls toward a thick horizontal hard-steel plate, striking it at 10.0 m/s. Given that the collision is completely elastic, at what speed will the sphere rebound?

59. [I] A steel ball (call it ball-1) is fired at 20.0 m/s at an identical steel ball (call it ball-2). What is the speed of each ball immediately after the collision if no kinetic energy is lost?

60. [I] A glass ball of mass 200 g is traveling directly at a steel ball of 400 g. An observer at rest in the lab sees the glass ball moving east at 10.0 m/s and the steel ball moving west at 20.0 m/s. What will be the speed at which the two balls approach or recede from each other after undergoing an elastic collision?

61. [III] A 1263-kg (i.e., 2785-lb) Triumph TR-8 sports car traveling south at 40 km/h (i.e., $40 \times 0.2778 \text{ m/s}$) crashes head-on into a 1742-kg (i.e., 3840-lb) Checker cab moving north at 90 km/h (i.e., $90 \times 0.2778 \text{ m/s}$). If the two cars remain tangled together but free to coast, describe their motion immediately after the collision. What is their final speed? How much kinetic energy is lost?

SOLUTION: Momentum is conserved for the Triumph (T) and cab (C).

Take south to be positive, in which case $v_{Ti} = (40 \times 0.2778 \text{ m/s})$ and $v_{Ci} = -(90 \times 0.2778 \text{ m/s})$. The collision is totally inelastic. Initially, $p_{Ci} = m_C v_{Ci}$ and $p_{Ti} = m_T v_{Ti}$. Finally, $p_f = (m_C + m_T) v_f$ and since $p_i = p_f$, $m_C v_{Ci} + m_T v_{Ti} = (m_C + m_T) v_f$. Putting the numbers in $(1263 \text{ kg})(11.1 \text{ m/s}) + (1742 \text{ kg})(-25.0 \text{ m/s}) = (3005 \text{ kg}) v_f$ and $v_f = -9.8 \text{ m/s}$. $KE_i - KE_f = \frac{1}{2} m_T v_{Ti}^2 + \frac{1}{2} m_C v_{Ci}^2 - \frac{1}{2} (m_T + m_C) v_f^2 = 4.8 \times 10^5 \text{ J}$.

62. [III] In an arrangement for measuring the muzzle velocity of a rifle or pistol, the bullet is fired up at a wooden mass, into which it imbeds. The wood is blasted straight up into the air to a measured height h . Assuming negligible losses to friction, write an expression for the velocity in terms of the known masses and height. If a 100-grain (6.48-g) 25-06 Remington rifle bullet is fired into a 5.00-kg block that then rises 4.0 cm into the air, what was the muzzle speed of that bullet?

63. [II] An 8.0-kg puck floating on an air table is traveling east at

15 cm/s. Coming the other way at 25 cm/s is a 2.0-kg puck on which is affixed a wad of bubble gum. The two slam head-on into each other and stick together. Find their velocity after the impact. How much kinetic energy is lost?

64. [II] Two identical blocks, each of mass 10.0 kg, are to be used in an experiment on a frictionless surface. The first is held at rest on a 20.0° inclined plane 10.0 m from the second, which is at rest at the foot of the plane. The one descends the incline, slams into and sticks to the second, and they sail off together horizontally. Calculate their speed immediately after impact.

65. [II] A 2000-kg car is traveling east at 20.0 m/s when it's rammed in the rear by a 1000-kg car that was traveling at 30.0 m/s just before impact. The two cars tangle together and move off together at a speed that we now wish to determine—please do so.

66. [II] As seen from the window of a space station, a 100-kg satellite sailing along at 10.0 m/s collides head-on with a small 300-kg asteroid, which was initially at rest. Taking the collision to be elastic, and neglecting their mutual gravitational interaction, what are the final velocities of the two bodies? What were their "relative speeds" before and after the collision?

67. [II] Two billiard balls, one heading north at 15.0 m/s and one heading south at 10 m/s, collide head-on. Take the collision to be perfectly elastic. What is the post-impact speed of each ball? [Hint: It will help if you remember that for completely elastic collisions, the "relative speeds" before and after are equal.]

68. [II] A skater with a mass of 75 kg is traveling east at 5.0 m/s when he collides with another skater of mass 45 kg heading 60° south of west at 15 m/s. If they stay tangled together, what is their final velocity?

69. [III] A soft clay block is suspended so as to form a so-called ballistic pendulum, as shown in Fig. P69. A bullet is fired point-blank into the block, imbedding itself therein and raising the latter to a height h . Write an expression for the muzzle speed of the bullet in terms of g , h , and the masses. Remember that although friction on the pendulum is negligible, clay-bullet friction losses are not, and *mechanical energy is not conserved* in the collision.



Figure P69

70. [III] Referring to Problem 69, derive an expression for the percentage of the kinetic energy converted into internal energy during the bullet-clay impact.

71. [III] Two identical hard spheres, one at rest and the other moving, slam into one another in a *non-head-on* elastic collision, as shown in Fig. P71. Prove that they will always fly apart at 90° to each other.



Figure P71

72. [III] Earlier (p. 223) we saw that $v_{1i} = v_{2f} - v_{1f}$ for elastic collisions where body-2 was initially at rest. To show that this is true in general, first establish that $m_1(v_{1i} + v_{1f})(v_{1i} - v_{1f}) = m_2(v_{2f} + v_{2i})(v_{2f} - v_{2i})$; then obtain $m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$ and finally show that $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$.