Chapter 6, Part 2 and Chapter 12

Notes: in #62, Earth's distance from Sun is 149,600,000 km
in #63, Moon's mass is 7.35 x 10^22 kg

5. [I] A 25-g pebble is stuck in the tread of a 28-in.-diameter tire. If the tire can exert an inward radial friction force of up to 20 N on the pebble, how fast will the pebble be traveling with respect to the center of the wheel when it flies out tangentially? [*Hint: We have the mass, radius, and centripetal force, and need the speed. The defining equation for F_C should do the trick.*]

6. [I] Determine the maximum speed that can be reached by a 19.61-N (as measured on Earth) steel ball tied to the end of a thin thread. The ball, which is essentially weightless, is being whirled in a circle of 2.00-m radius by an astronaut out in space. The thread has a breaking strength of 16.0 N.

7. [I] A 10.0-kg mass is tied to a 3/16-in. Manila line, which has a breaking strength of 1.80 kN. What is the maximum speed the mass can have if it is whirled around in a horizontal circle with a 1.0-m radius and the rope is not to break?

8. [I] A youngster on a carousel horse 5.0 m from the center revolves at a constant rate, once around in 15.0 s. What is her acceleration?

9. [I] A baseball player rounds second base in an arc with a radius of curvature of 4.88 m at a speed of 6.1 m/s. If he weighs 845 N, what is the centripetal force that must be acting on him? Notice how this limits the tightness of the turn. What provides the centripetal force?

10. [I] Compute the Earth's centripetal acceleration toward the Sun. Take the time it takes to go once around as 365 d and the radius on average as 1.50×10^8 km.

SOLUTION: The centripetal acceleration is given by $a_C = v^2/r$, but we don't know v. Still, $v = l/t$ and we can compute l, which is the circumference. $l = 2\pi r = 2\pi(1.50 \times 10^{11}\text{ m}) = 9.425 \times 10^{11}$ m. The time to go once around, the period, is (365 d)(24 h/d)(60 min/h)(60 s/min) = 3.154×10^7 s. Thus $v = l/t = (9.425 \times 10^{11}\text{ m})/(3.154 \times 10^7\text{ s}) = 2.988 \times 10^4$ m/s. And finally, $a_C = v^2/r = (2.988 \times 10^4\text{ m/s})^2/(1.50 \times 10^{11}\text{ m}) = 5.95 \times 10^{-3}\text{ m/s}^2$.

11. [I] A hammer thrower at a track-and-field meet whirls around at a rate of 2.0 revolutions per second, revolving a 16-lb ball at the end of a cable that gives it a 6.0-ft effective radius. Compute the inward force that must be exerted on the ball.

12. [I] A test tube in a centrifuge is pivoted so that it swings out horizontally as the machine builds up speed. If the bottom of the tube is 150 mm from the central spin axis, and if the machine hits 50 000 revolutions per minute, what would be the centripetal force exerted on a giant amoeba of mass 1.0×10^{-8} kg at the bottom of the tube?

13. [I] A 100-mm-long test tube is held rigidly at 30° with respect to the vertical in a centrifuge with its top lip 5.0 cm from the central spin axis of the machine and its bottom somewhat farther. If it rotates at 40 000 revolutions per minute, what is the centripetal acceleration of a cell at the bottom of the tube?

14. [II] A circular automobile racetrack is banked at an angle θ such that no friction between road and tires is required when a car travels at 30.0 m/s. If the radius of the track is 400 m, determine θ.

15. [II] A front-loading clothes washer has a horizontal drum that is thoroughly perforated with small holes. Assuming it to spin dry at 1 rotation per second, have a radius of 40 cm, and contain a 4.5-kg wet Teddy bear, what maximum force is exerted by the wall on the bear? What happens to the water?

16. [II] EXPLORING PHYSICS ON YOUR OWN: Drop a string of length L through the hole in a vertically held spool of thread (Fig. P16). Tie 10 paper clips to each end and twirl the string so that one bunch of clips moves in a horizontal circle at a speed v while the other hangs vertically. Neglecting friction, write an expression (a) for the distance d as a function of v and L where $L = (r + d)$ and (b) for g in terms of v and r, the radius of the circle.

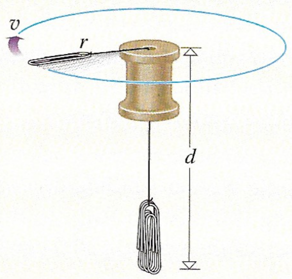

Figure P16

17. [II] A skier, of mass m, comes down a slope that has the shape of a vertical segment of a circle (of radius r, with the center above the path), ending in a tangential, flat, horizontal run. Write an expression in terms of v, m, r, and g for the normal force exerted by the snow on the skis at the bottom just before the skier leaves the circular portion. [*Hint: At the bottom of the arc the ground must supply the centripetal force and support the weight.*]

18. [II] While whirling around in a vertical circle with a radius of 1.50 m, a 2.00-kg mass is held on a rope attached to a very light spring scale. What value does the scale read when the mass is moving at 4.00 m/s at the lowest point of its orbit?

19. [II] A scale is fitted into the seat of a roller coaster car, and a person weighing 800 N sits down on it. The car then descends along a path that has the shape of a 100.0-m-radius vertical circle with its lowest point at the bottom where the car reaches its greatest speed of 40.0 m/s. What is the maximum reading of the scale?

20. [II] THIS PROBLEM DEALS WITH CENTRIPETAL ACCELERATION. An object moves in a circular orbit at a constant speed. It takes a time T, called the *period*, to go once around. We want to show that

$$a_C = \frac{4\pi^2 r}{T^2}$$

But first, (a) write an expression for the circumference of the circular orbit. (b) Knowing how long it takes to go once around, write an expression for the orbital speed in terms of r and T. (c) Now derive the above expression for a_C.

21. [II] After a few seconds a toy car on a 2.00-m diameter circular track reaches and thereafter maintains a constant speed. If it then makes 1200 revolutions in 1.00 h (a) How fast is it moving? (b) What is its centripetal acceleration?

22. [II] A 1000-kg car traveling on a road that runs straight up a hill reaches the rounded crest at 10.0 m/s. If the hill at that point has a radius of curvature (in a vertical plane) of 50 m, what is the net downward force acting on the car at the instant it is horizontal at the very peak?

SOLUTION: The net downward force equals the net upward force which is the normal force. On a flat road $F_W = F_N$, but here at the top of a vertical curved road there must be a net downward force equal to F_C; otherwise the car would fly off tangentially. Thus, taking down as positive, $F_W - F_N = F_C$. The centripetal force is given by $F_C = ma_C = mv^2/r$ and so $F_N = F_W - F_C = mg - mv^2/r = (1000\text{ kg})(9.81\text{ m/s}^2) - (1000\text{ kg})(10.0\text{ m/s})^2/(50\text{ m}) = 7.8$ kN.

23. [II] At a given instant, someone strapped into a roller coaster car hangs upside down at the very top of the circle (of radius 25.0 m) while executing a so-called loop-the-loop. At what speed must he be traveling if at that moment the force exerted by his body on the seat is half his actual weight? Assume that at the start of the ride the straps were fairly loose.

24. [II] Suppose you wish to whirl a pail full of water in a vertical circle without spilling any of its contents. If your arm is 0.90 m-long (shoulder to fist) and the distance from the handle to the surface of the water is 20.0 cm, what minimum speed is required?

25. [II] A cylindrically shaped space station 1500 m in diameter is to revolve about its central symmetry axis to provide a simulated 1.0-*g* environment at the periphery. (a) Compute the necessary spin rate. (b) How would "*g*" vary with altitude up from the floor (which is the inside curved wall of the cylinder)?

26. [II] A stunt pilot flying an old biplane climbs in a vertical circular loop. While upside down, the force acting upward normally on the seat is one third of her usual weight. If the plane is traveling at that moment at 300 km/h, what is the radius of the loop? (A World War I pilot did this little trick without fastening his seat belt and—you guessed it—fell out. What can you say about his speed at the top of the loop?)

27. [III] Take the Earth to be a perfect sphere of diameter 1.274×10^7 m. If an object has a weight of 100 N while on a scale at the South Pole, how much will it weigh at the Equator? Take the equatorial spin speed to be $v = 465$ m/s.

28. [III] Design a carnival ride on which standing passengers are pressed against the inside curved wall of a rotating vertical cylinder. It is to turn at most at $\frac{1}{2}$ revolution per second. Assuming a minimum coefficient of friction of 0.20 between clothing and wall, what diameter should the ride have if we can safely make the floor drop away when it reaches running speed?

Miscellaneous data: $M_\oplus = 5.975 \times 10^{24}$ kg, $M_\odot = 1.987 \times 10^{30}$ kg, $M_☾ = 7.35 \times 10^{22}$ kg, $R_\odot = 6.97 \times 10^8$ m, $R_\oplus = 6371.23$ km, $R_☾ = 1.74 \times 10^6$ m, $r_{\odot\oplus} = 1.495 \times 10^{11}$ m, $r_{\oplus ☾} = 3.844 \times 10^8$ m.

SECTION 5.3: THE LAW OF UNIVERSAL GRAVITATION

SECTION 5.4: TERRESTRIAL GRAVITY

29. [I] What would happen to the weight of an object if its mass was doubled while its distance from the center of the Earth was also doubled?

30. [I] The gravitational attraction between a 20-kg cannonball and a marble separated center-to-center by 30 cm is 1.48×10^{-10} N. Compute the mass of the marble.

31. [I] Suppose that two identical spheres, separated center-to-center by 1.00 m, experience a mutual gravitational force of 1.00 N. Compute the mass of each sphere.

32. [I] At what center-to-center distance from the Earth would a 1.0-kg mass weigh 1.0 N?

33. [I] Suppose the Earth were compressed to half its diameter. What would happen to the acceleration due to the gravity at its surface?

34. [I] Compare the gravitational force of the Earth on the Moon to that of the Sun on the Moon.

35. [I] If the average distance between Uranus (♅) and Neptune (♆) is 4.9×10^9 km, and $M_♅ = 14.6 M_\oplus$ while $M_♆ = 17.3 M_\oplus$, compute their average gravitational interaction.

36. [I] Imagine two uniform spheres of radius R and density ρ in contact with each other. Write an expression for their mutual gravitational interaction as a function of R, ρ, and G.

37. [I] If you can jump 1.00 m high on Earth, how high can you jump on Venus, where $g_♀ = 0.88 g_\oplus$? Assume the same takeoff speed.

38. [I] What fraction of what you weigh on Earth would you weigh in a rocket ship firing its rockets so that it was stationary with respect to the center of the planet $4R_\oplus$ from its surface?

SOLUTION: The ship is five Earth-radii from the center of the Earth. Given that your weight on Earth is F_W, since the force drops off as the distance square, your weight in the ship is $F_W/25$.

39. [I] Consider two subatomic particles, an electron and a proton, which have masses of 9.1×10^{-31} kg and 1.7×10^{-27} kg, respectively. When separated by a distance of 5.3×10^{-11} m, as they are in a hydrogen atom, the electrical attraction (F_E) between them is 8.2×10^{-8} N. Compare this with the corresponding gravitational interaction. How many times larger is F_E than F_G?

40. [I] Considering the Earth as a sphere, when does $g_0 = g_\oplus$? Which of these is a constant and which is a function of r? What would have to happen for $g_0 = g$?

41. [I] The acceleration due to gravity on the surface of Mars is 3.7 m/s². If the planet's diameter is 6.8×10^6 m, determine the mass of the planet and compare it to Earth.

42. [I] THIS PROBLEM IS ABOUT THE ACCELERATION DUE TO GRAVITY. A spacecraft of mass m is at a distance of $4R_\oplus$ from the center of the planet. (a) What is the formula for the gravitational force on the vehicle at that distance? (b) What is the weight of the vehicle at a distance of $4R_\oplus$? (c) Alternatively, express that weight in terms of the acceleration due to gravity at $4R_\oplus$. (d) Write an expression for the gravitational acceleration at $4R_\oplus$, in terms of $R_\oplus$, G, and $M_\oplus$.

43. [II] What is the value of the acceleration due to the Moon's gravity 100 km above its surface?

44. [II] Imagine an astronaut having a mass of 70 kg floating in space 10.0 m away from the center-of-gravity (the point where all the mass may be imagined to act gravitationally, p. 249) of an *Apollo* Command Module whose mass is 6.00×10^3 kg. Determine the gravitational force acting on, and the resulting accelerations (at that instant) of both the ship and the person.

45. [II] Venus has a diameter of 12.1×10^3 km and a mean density of 5.2×10^3 kg/m³. How far would an apple fall in one second at its surface? [*Hint: We need the acceleration due to gravity at Venus's surface, and to get that we must first determine the planet's mass.*]

46. [II] Imagine a great sphere of water (of density 1.00×10^3 kg/m³) floating in space. If it has a radius of 10.0×10^3 km (about 1.57 times the size of Earth), what would be the acceleration due to gravity at its surface? Check your answer using the Earth's mean radius (6.35×10^6 m) and density (5.5×10^3 kg/m³).

47. [II] Gold has density of 19.3×10^3 kg/m³. How big would a solid gold sphere have to be if the acceleration due to gravity at its

surface is to be 9.81 m/s^2? Check your answer against the radius of the Earth, which has a mean density of $5.5 \times 10^3\ kg/m^3$.

48. [II] Given that $M_☾/M_\oplus = 0.012\,30$ and $R_☾/R_\oplus = 0.273\,1$, compute the ratio of an astronaut's Moon-weight ($F_{W☾}$) to Earth-weight ($F_{W\oplus}$).

SOLUTION: $F_{W☾}/F_{W\oplus} = (GM_☾ m/R_☾^2)/(GM_\oplus m/R_\oplus^2) = (M_☾/M_\oplus)/(R_☾/R_\oplus)^2 = 0.012\,30/(0.273\,1)^2 = 0.1649$.

49. [II] Taking the surface value of $g_\oplus$ (see p. 153) to be g_0, show that

$$g_\oplus = g_0(R_\oplus/r)^2 \quad \text{for } r \geq R_\oplus$$

50. [II] Locate the position of a spaceship on the Earth-Moon center line such that, at that point, the tug of each celestial body exerted on it would cancel and the craft would literally be weightless.

51. [II] Mars has a mass of $M_♂ = 0.108 M_\oplus$ and a mean radius $R_♂ = 0.534 R_\oplus$. Find the acceleration of gravity at its surface in terms of $g_o = 9.8\ m/s^2$.

52. [II] Three very small spheres of mass 2.50 kg, 5.00 kg, and 6.00 kg are located on a straight line in space away from everything else. The first one is at a point between the other two, 10.0 cm to the right of the second and 20.0 cm to the left of the third. Compute the net gravitational force it experiences.

53. [II] It is believed that during the gravitational collapse of certain stars, such great densities and pressures will be reached that the atoms themselves will be crushed, leaving only a residual core of neutrons. Such a *neutron star* is, in some respects, very much like a giant atomic nucleus with a tremendous density of roughly about $3 \times 10^{17}\ kg/m^3$. Compute the surface acceleration due to gravity for a one-solar-mass neutron star.

54. [III] Figure P54 shows two concentric, thin, uniform spherical shells of mass m_1 and m_2, at the center of which is a small ball of lead of mass m_3. Write an expression for the gravitational force exerted on a particle of mass m at each point A, B, and C located at distances r_A, r_B, and r_C from the very center. [*Hint: There is no force inside a spherical mass shell due to that mass.*]

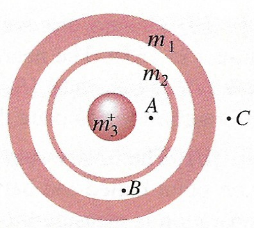

Figure P54

55. [III] Write an expression for the gravitational force on a small mass m imbedded in a uniform spherical cloud of mass M and radius R. Take the particle to be at $r < R$. [*Hint: There is no force inside a spherical mass shell due to that mass.*]

56. [III] Two 2.0-kg crystal balls are 1.0 m apart. Compute the magnitude and direction of the gravitational force they exert on a 10-g marble located 0.25 m from the center-to-center line as shown in Fig. P56.

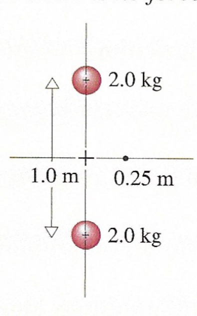

Figure P56

57. [III] A neutron star (see Problem 53) can be envisioned as an immense nucleus held together by its self-gravitation. What is the shortest period with which such a star could rotate if it's not to lose mass flying off at the equator? Take $\rho = 1 \times 10^{17}\ kg/m^3$. It is widely believed that *pulsars*, strange celestial emitters of precisely pulsating radiation, are rapidly rotating neutron stars (see p. 261).

58. [III] Draw a graph of the weight of an object of mass m due to the Earth versus the height above the surface, out to roughly 700 km. What can you say about the curve (as long as $R >> h$)?

59. [III] Let M be the mass of a uniform spherical planet of radius R. If h is the height above its surface, show that the absolute gravitational acceleration g_P varies with h as

$$g_P = \frac{GM}{R^2}\left(1 + \frac{h}{R}\right)^{-2}$$

This expression can be approximated using the binomial expansion. {See **MATH REVIEW: PART A-5** on the **CD**.}

$$(a + x)^n = a^n + na^{n-1}x + \tfrac{1}{2}n(n-1)a^{n-2}x^2 + \cdots$$

where $x^2 < a^2$. Here, $a = 1$, $x = h/R$, $n = -2$, and we limit the calculation to the case where $h << R$. Show that

$$g_P \approx \frac{GM}{R^2}\left(1 - \frac{2h}{R}\right)$$

Notice that GM/R^2 is the surface value of g_P occurring when $h = 0$, as in Eq. (5.6).

60. [III] Calculate the acceleration due to gravity 10 000 m above the Earth's surface in the following two ways: (1) using $g_\oplus = GM_\oplus/r^2$; and (2) using the approximation of Problem 59. Compare your results.

61. [III] In light of Problem 59, determine the acceleration due to gravity experienced by the Lunar Module when it was 100 m above the Moon's surface. Is it appreciably different from the surface value?

SECTION 5.5: THE LAWS OF PLANETARY MOTION

62. [I] Using the data for the Earth's orbit, compute the mass of the Sun.

63. [I] Determine the approximate speed of a Lunar Orbiter revolving in a circular orbit at a height of 62 km. Take the Moon's radius as 1738 km.

64. [I] Each of the *Apollo* Lunar Modules was in a very low orbit around the Moon. Given a typical mass of 14.7×10^3 kg, assume an altitude of 60.0 km and determine the orbital period.

65. [I] For any Earth satellite in a circular orbit, show that its period (in seconds) and its distance from the center of the planet (in meters) are related by way of

$$T = 3.15 \times 10^{-7}(r_\oplus)^{\frac{3}{2}}$$

66. [I] *Sputnik I*, the first artificial satellite to circle the planet (October 1957) had a mean orbital radius of 6950 km. Compute its period.

67. [I] A satellite is to be raised from one circular orbit to another twice as large. What will happen to its period?

68. [I] Referring to Problem 61, compare the two orbital speeds v_1 and v_2.