pletely at rest. The scale reads 90.0 N and the hanging mass on the right weighs 45.0 N . (a) Draw a free-body diagram of the beam. (b) How much does the mass on the left weigh? (c) Determine the weight of the beam.

Figure P76

77. [I] An $8000-\mathrm{N}$ automobile is stalled one-quarter of the way across a bridge (see Fig. P77). Compute the additional reaction forces at supports $A$ and $B$ due to the presence of the car. Take the length of the bridge to be $\overline{A B}$.

SOLUTION: Let $L$ be the length of the bridge;

$$
\text { Eq.(i) }+\uparrow \sum F_{y}=0=F_{\mathrm{RA}}+F_{\mathrm{RB}}-8000 \mathrm{~N}
$$

$$
\text { Eq.(ii) } \quad c+\sum \tau_{\mathrm{A}}=0=(8000 \mathrm{~N})\left(\frac{3}{4} L\right)-F_{\mathrm{RB}} L
$$

Using Eq.(ii), $F_{\mathrm{RB}}=(8000 \mathrm{~N}) \frac{3}{4}=6000 \mathrm{~N}$ and using Eq.(i)

$$
F_{\mathrm{RA}}=2000 \mathrm{~N}
$$

Figure P77
78.[I] The little bridge in Fig. P78 has a weight of 20.0 kN , which acts at its center. Calculate the reaction forces at points $A$ and $B$.


Figure P78
79. [I] The scale in Fig. P79 reads 2.1 N. Neglecting the weight of the beam and assuming the pivot, which is in the first hole on the left (see Fig. Q24), is frictionless, determine the hanging load.
80. [I] With the previous problem in mind, completely specify the reaction force exerted on the beam by the pivot.
81. [I] This problem deals with an object in equilibrium. Each of the hanging objects on the right in Fig. P81 weighs 1.0 N. Assume the pivot (which is in the notch at the far left end) is frictionless. If the weight of the uniform beam is neglected, and the scale reads 5.0 N , (a) write an expression for the sum of the torques about the pivot and set it equal to zero. (b) What is the value of the angle $\theta$ ? (c) What is the value of the horizontal component of the reaction force on the pivot? (d) What is the value of the vertical component of the reaction force on the pivot?

Figure P79

82. [II] Determine the magnitude and direction of the reaction force of the pivot on the beam in the previous problem. Refer to the previous problem. Notice that the pivot rod does not go through a whole, but instead sits in a half-circle groove at the end of the beam. Explain how the beam stays in place.
83. [II] Two campers carry their gear ( 90.72 kg ) on a light, rigid horizontal pole whose ends they support on their shoulders 1.829 m apart. If Selma experiences a compressive force of 533.8 N, where is the load hung on the pole, and what will Rocko feel?
84. [II] The weightless ruler in Fig. P84 is in equilibrium. Determine both the unknown mass $M$ and the left-hand scale reading.

Figure P84

85.[II] If the beam in Fig. P85 is of negligible mass, what value does the scale read (in kg )?

86. [II] Figure P86 shows a laboratory model of a roof truss constructed of two very light rods pinned at points $B$ and $C$. Because of this construction, the forces act only along the lengths of the rods.
A light cable tightened by a turnbuckle runs from $A$ to $C$ to complete the truss. What do the two spring scales read in newtons? Determine the compression in rod $A B$. [Hint: Find out the reading of scale-1 and then draw a free-body diagram of A.]
87. [II] The little temporary bridge in Fig. P87 consists of two very light planks. The $480-\mathrm{N}$ person stands 3.00 m from the end of the upper board, which itself rests on rollers. What are the reaction forces at points $A, C$, and $D$ ?


Figure P86


Figure P87
88. [II] Find the force $F_{\mathrm{R}}$ in Fig. P88 by taking torques about both point $A$ and the point where the lines-of-action of the two tensile forces on the ropes intersect.
89. [II] The hand in Fig. P89 exerts a force of 300 N on the hammer handle. Compute the force acting on the nail, just as the head begins to pivot about its front edge in contact with the board.
90. [II] A forceps is used to pinch off a piece of rubber tubing, as shown in Fig. P90. What is the force exerted on the rubber if each finger squeezes with 10.0 N ? What is the force exerted on the pivot by each half of the forceps?
91. [III] Suppose a downward force of 80 N acts on the pedal of the bike shown in Fig. P91. The chain goes around the front chain wheel (of radius 10 cm ) and the rear sprocket wheel (of radius 3.0 cm ). The pedal is attached to a $17-\mathrm{cm}$ crank arm that turns around point $A$. What impelling force will the scale read (the bike, chain,

(a)


Figure P88
(b)

Figure P89


Figure P90

and wheel are motionless at the moment depicted when the crank arm is horizontal)? What provides that force?

Figure P91

92. [III] A sphere of mass 10.0 kg rests in a groove, as shown in Fig. P92. Assuming no friction and taking the weight of the sphere to act at its center, compute the reaction forces exerted by the two surfaces.

Figure P92

93. [III] Determine the internal forces acting at point $A$ in Fig. P93 when a $1000-\mathrm{N}$ weight is hung from the hook. All of the structural members can be taken as weightless. [Hint: Because of the pinned construction, the forces act only along the lengths of the rods.]

Figure P93

94. [III] Figure P94 shows a smooth rod resting horizontally inside a bowl. Compute the force $F$ that will maintain the rod in position, given negligible friction.

Figure P94


## SECTION 8.7: EXTENDED BODIES \& THE CENTER-OF-GRAVITY.

95. [I] A uniform cylindrical broom handle is 2.0 m long and weighs 8.0 N. Locate its center-of-gravity.
96. [I] Two identical $60-\mathrm{cm}$ diameter hollow copper spheres are attached to each other with superglue. Locate the center-of-gravity of the system.
97. [I] Two identical uniform $50-\mathrm{cm}$ long rods are attached to each other at their centers so that they lie in the same plane making an angle of $90^{\circ}$ like $\mathrm{a}+$ sign. One of four identical $10-\mathrm{N}$ lead spheres is then fixed at each end of the rods. Locate the center-of-gravity of the system.
98. [I] A $40-\mathrm{cm}$ diameter 20-N metal sphere is glued to the surface of a $120-\mathrm{cm}$ diameter $20-\mathrm{N}$ plastic sphere. Locate the center-ofgravity of the system.
99. [I] Two lead spheres weighing 20.0 N and 10.0 N are separated 30.0 cm , center-to-center, by a horizontal weightless rigid rod. Locate the center-of-gravity of the system.

> SOLUTION: Place the spheres on the positive $x$-axis with the origin at the center of the $20-\mathrm{N}$ sphere. Using Eq. ( 8.27 ), with the total weight being $30.0 \mathrm{~N}, x_{\mathrm{cg}}(30.0 \mathrm{~N})=(10.0 \mathrm{~N})(0.300 \mathrm{~m})$ and $x_{\mathrm{cg}}=0.100 \mathrm{~m}$. The c.g. is 10.0 cm to the right of the center of the larger sphere.
100. [I] This problem deals with the center-of-gravity of a COMPOUND SYSTEM. To each end of a lightweight (horizontal) straight rod is affixed a horizontal metal ring. Each end of the rod is attached to the outer rim of a ring and the rod lies along the line connecting the centers of the two rings. One ring weighs 60.0 N , the other 15.0 N . (a) Locate the $c . g$. of each ring. (b) If their center-to-center separation is 100 cm , locate the center-of-gravity of the system.
101. [I] Two uniform blocks of wood $4 \mathrm{~cm} \times 4 \mathrm{~cm} \times 12 \mathrm{~cm}$, each weighing 1.0 N , are glued together as shown in Fig. P101. Find the c.g. of the system.
102. [I] A uniform piece of soft, thin copper wire $60-\mathrm{cm}$ long and weighing 1.2 N is bent into three equal-length segments to form two right angles, so that it has the shape of an angular letter "C." Locate the folded


Figure P101 wire's c.g.
103. [I] A $0.50-\mathrm{m}$-long thin steel rod with a mass per unit length of $0.50 \mathrm{~kg} / \mathrm{m}$ is bent in half (making a right angle) into the shape of a letter "L." Locate its c.g.
104. [I] The glider in Fig. P104 is descending at a constant speed. If the drag $\left(F_{\mathrm{D}}\right)$ is 600 N and the plane weighs 6000 N , determine both the angle of descent, $\theta$, and the lift, $F_{\mathrm{L}}$.


Figure P104
105. [I] Scale-1 in Fig. P105 reads 100 N. What does scale-2 read? How much does the suspended body weigh? Discuss the location of the vertical line passing through the $c . g$.


Figure P105
106. [I] Refer to the mobile in Fig. P106. If the star weighs 10.0 N , how much does the sphere weigh?


Figure P106
107. [I] Now suppose we displace the sphere in Problem 106 downward, whereupon the top rod tilts down at $30.0^{\circ}$ with the horizontal. Will the mobile be in equilibrium?
108. [I] This problem explores rotational equilibrium. The heavy motionless uniform steel beam hanging in Fig. P108 weighs
an unspecified amount $F_{\mathrm{w}}$. (a) Where does the weight of the beam act?
(b) Draw the free-body diagram of the beam.
(c) What is the value of the net upward force due to the supporting ropes? (d) Which two forces produce torques about


Figure P108 the left end of the beam? (e) Determine the tension in each rope in terms of $F_{\mathrm{w}}$.
109. [I] This problem explores rotational equilibrium. The uniform beam in Fig. P109 weighs 0.8 N , and the scale reads 2.5 N . Assume the pivot in the first hole at the right (see p. 269) is frictionless. (a) Where does the weight of the beam act? (b) Draw the free-body diagram of the beam. (c) List all the forces acting on the beam and specify the directions (clockwise or counterclockwise) of the torques they produce about the pivot. (d) What is the tension in the rope at the far left? (e) Write an expression for the sum-of-the-torques about the pivot. (f) Determine the net weight of the hanging load.

## Figure P109


110. [I] This problem explores rotational equilibrium. The heavy motionless uniform beam shown in Fig. P110 is frictionlessly pivoted at its left end. (a) In what directions are the horizontal and vertical reaction forces on the pivot? Explain why. (b) Draw the free-body diagram of the beam. (c) List all the forces acting on the beam and specify the directions (clockwise or counterclockwise) of the torques they produce about the pivot. (d) If the beam has the same weight $\left(F_{\mathrm{w}}\right)$ as the load it supports, what is the value of the vertical reaction force at the pivot, in terms of $F_{\mathrm{w}}$ ? (e) What is the value of the vertical component of the tension? [Hint: To find the direction of the reaction force at the pivot (without making any calculations), imagine that the pivot pin was removed; the beam would then move left while simultaneously rotating clockwise around the end of the rope.]


Figure P110
111. [I] The tilted motionless uniform heavy rod shown in Fig. P111 is frictionlessly pivoted at its right end. Draw the free-body diagram of the rod. [Hint: To find the direction of the reaction force at the pivot (without making any calculations), imagine that the pivot pin was removed; the upper portion of the rod would rotate counterclockwise around the end of the rope.]
112. [II] A pole vaulter carries a $6.0-\mathrm{m}$ uniform pole horizontally. He holds one end with his right hand pushing downward and 1.0 m away pushes upward with his left hand. If the pole weighs 40 N , compute both forces he exerts. What is the net force


Figure P111 he applies? What would these forces be if the pole were vertical? What does this answer suggest about the carrying angle?
113. [II] We wish to stack five uniform wooden blocks so that they extend as far right as possible and still remain stable. How should each be positioned? Can the top block have its entire length beyond the edge of the bottom block (Fig. P113)?

Figure P113

114. [II] This problem deals with rotational equilibrium. We want to compute $\theta$, the smaller angle between the string and the beam, in the photo shown in Fig. P114. Each of the hanging objects weighs 1.0 N , and the scale reads 4.6 N . Assume the pivot, in the fifth hole from the right, is frictionless and the weight of the uniform beam is 0.80 N . (a) Where does the weight of the beam act? (b) What are the horizontal and vertical forces exerted on the beam by the scale? (c) What is the value of the horizontal component of the reaction force at the pivot. (d) List all the forces acting on the beam and specify the directions (clockwise or counterclockwise) of the torques they produce about the pivot. (e) Find $\theta$.

Figure 114

115. [II] This problem deals with rotational equilibrium. We want to find the reaction force at the pivot in the photo shown in

Fig. P81. Each of the hanging objects on the right weighs 1.0 N and the scale reads 5.0 N . Assume the pivot is frictionless and the weight of the uniform rod is 0.8 N . (a) Where does the weight of the rod act? (b) In what directions are the horizontal and vertical reaction forces on the pivot? Explain why. (c) Draw the free-body diagram of the rod. (d) List all the forces acting on the rod and specify the directions (clockwise or counterclockwise) of the torques they produce about the pivot. (e) Compute $\theta$, this time including the weight of the rod. (f) What is the value of the vertical component of the reaction force at the pivot? (g) What is the value of the horizontal component of the reaction force at the pivot? Refer to Problem 81.
116. [II] An essentially weightless $10.0-\mathrm{m}$-long beam is supported at both ends, as in Fig. P116. A $300-\mathrm{N}$ child stands 2.00 m from the left end, and a $6.00-\mathrm{m}$-long stack of newspapers weighing 100 N per linear meter is uniformly distributed at the other end. Determine the two reaction forces supporting the beam.

117. [II] Figure P117 shows two ropes supporting a body. (a) Prove that the resultant of the two tensile forces, whose magnitudes are indicated by the scale readings, has the same magnitude as the weight. (b) Show that the plumb line passes through the body's c.g.

[^0]
118. [II] A $55.0-\mathrm{kg}$ woman is standing "rigid"-straight upright, hands at sides, feet together. At their widest, her two feet have a breadth of 20.0 cm and her $c . g$. is 95.0 cm above the ground. What minimum horizontal force in the plane of her body (i.e., perpendicular to the forward direction) will tilt her over if it acts along a shoulder-to-shoulder line $145-\mathrm{cm}$ high?
119. [II] This problem examines the forces acting on an object in rotational equilibrium. A heavy uniform ladder leans motionlessly against a smooth, essentially frictionless wall. It rests on a rough floor that keeps it from sliding out. (a) Identify the two components of the reaction force of the floor on the ladder. (b) Identify the single reaction force of the wall on the ladder. (c) What is the relationship between the friction force at the ground and the normal force exerted by the wall? (d) Draw the free-body diagram of the ladder.
120. [II] This problem examines the forces acting on an object in rotational equilibrium. A 200-N uniform board leans motionlessly with its high end against a smooth, essentially frictionless, wall. The other end rests on a rough floor that keeps it from sliding out. (a) With the previous problem in mind, draw the free-body diagram of the board. (b) Given that the board-floor static coefficient of friction is 0.50 , show that the reaction force of the wall must be $\leq 100 \mathrm{~N}$ if the board is not to slip. (c) Show that the minimum angle the board can make with the floor without sliding is $45^{\circ}$.
121. [II] An $8.0-\mathrm{m}-\mathrm{long}, 30-\mathrm{kg}$ uniform plank leans against a smooth wall, making an angle of $60.0^{\circ}$ with the ground. Compute the reaction force, $\overrightarrow{\mathrm{F}}_{\mathrm{RG}}$, of the ground on the plank.
122. [II] In Problem 74, Harry and Gretchen were sitting on a weightless seesaw. Suppose now that the beam is uniform and weighs 200 N -find the new pivot point.
123. [II] A $65-\mathrm{kg}$ woman is horizontal in a pushup position in Fig. P123. What are the forces acting on her hands and feet?

Figure P123

124. [II] A large box of breakfast cereal is $6.0-\mathrm{cm} \times 35-\mathrm{cm} \times 46-$ cm high and has a net mass of 1.0 kg . The contents are a uniformly distributed material. The box is next to an open window. Determine the speed of a uniform wind hitting the large surface if it just starts tilting the box over. The force exerted by a perpendicular wind in newtons on each square meter is given roughly by $0.6 v^{2}$, wherein $v$ is expressed in $\mathrm{m} / \mathrm{s}$. Assume the distributed force acts at the geometrical center of the face.
125. [III] One technique for measuring the $c . g$. of a person is illustrated in Fig. P125. The board is positioned according to the person's height ( $h$ ), and the scales are then reset to zero. The participant lies down, and the scale readings are taken. Determine an expression for $x_{\mathrm{cg}}$ in terms of the measured quantities.
126. [III] A uniform ladder leans against a smooth wall, making an angle of $\theta$ with respect to the ground. The ground's reaction force on the ladder is at an angle of $\phi$ with the horizontal. Determine the relationship between $\theta$ and $\phi$. Which is larger?


Figure P125
127. [III] The uniform plank in Fig. P127 is $5.00-\mathrm{m}$ long and weighs 100 N . The cord that attaches to the plank 1.00 m from the bottom end holds it from sliding, there being no friction. Find the tension in the cord.


Figure P127

## SECTION 8.8: TORQUE \& ROTATIONAL INERTIA

128. [I] Determine the moment-of-inertia of a solid disk 0.50 m in diameter, having a mass of 4.8 km .
129. [I] Consider a thin ring or hoop of mass $M$ and radius $R$ of negligible thickness. Compute the moment-of-inertia about its $c . m$. by imagining it to be made up of a large number of small segments. 130. [I] Two solid 3.0-kg cones, each with a base radius of 0.30 m , are glued with their flat faces together. Determine the moment-ofinertia of this device about the symmetry axis passing through both vertices.
130. [I] The Parallel-Axis Theorem states that if $I_{\mathrm{cm}}$ is the moment-of-inertia of a body about an axis through the c.m., then $I$, the moment-of-inertia about any axis parallel to that first one, is given by

$$
I=I_{\mathrm{cm}}+m d^{2}
$$

Here, $m$ is the object's mass, and $d$ is the perpendicular distance through which the axis is displaced. Accordingly, use the theorem for a thin rod of length $l ; I_{\mathrm{cm}}=\frac{1}{12} m l^{2}$ to compute $I$ about one end. 132. [I] Show that a small object (such as a sphere) that orbits a distant axis can be approximated by a point-mass when you want to compute its moment-of-inertia. Begin with the theorem of the previous problem.
133. [I] Envision a solid cylindrical wheel of radius $r$ and mass $m$ resting upright on its narrow edge on a flat plane. With Problem 101 in mind, compute the wheel's moment- of-inertia about the line of contact with the surface.
134. [I] Two small rockets are mounted tangentially on diametrically opposite sides of a cylindrically shaped artificial satellite. The spacecraft has a $1.0-\mathrm{m}$ diameter and a moment-of-inertia about its central symmetry axis of $25 \mathrm{~kg} \cdot \mathrm{~m}^{2}$. The rockets each develop a thrust of 5.0 N , are oppositely directed, and are aligned to produce a maximum spin-up of the craft. What's the resulting angular acceleration when they are both fired?
135. [II] A 26.0 -in.-diameter bicycle wheel supported vertically at its center is spun about a horizontal axis by a torque of 50.0 ft . lb . If the rim and tire together weigh 3.22 lb , determine the approximate angular acceleration of the wheel. Ignore the contribution of the spokes. Of course, these ugly units should cancel out. [Hint: The net applied torque equals the moment-of-inertia times the angular acceleration. This wheel is essentially a hoop.]
136. [II] Determine the angular acceleration that would result when a torque of $0.60 \mathrm{~N} \cdot \mathrm{~m}$ is applied about the central spin axis of a hoop of mass 2.0 kg and radius 0.50 m .
137. [II] How much torque must be applied to a hoop having a moment-of-inertia about its central symmetry axis of $1.50 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ if it is to uniformly accelerate from rest to $10.0 \mathrm{rad} / \mathrm{s}$ in 10.0 s ?
138. [II] Derive an expression for the acceleration of a hollow sphere of mass $m$ and radius $R$ rolling, without slipping, down an incline of angle $\theta$. Discuss the implications of the absence of $m$ and $R$ in this formula.
139. [II] A block of mass $m$ is tied to a light cord that is wrapped around a vertical pulley of radius $R$ and moment-of-inertia $I$. Assuming the bearings are essentially frictionless, write an expression for the rate at which the block will accelerate when released.
140. [II] A $10-\mathrm{kg}$ solid steel cylinder with a $10-\mathrm{cm}$ radius is mounted on bearings so that it rotates freely about a horizontal axis. Around the cylinder is wound a number of turns of a fine gold thread. A $1.0-\mathrm{kg}$ monkey named Fred holds on to the loose end and descends on the unwinding thread as the cylinder turns. Compute Fred's acceleration and the tension in the thread.
141. [II] How would Fred manage in Problem 140 if the bearings exerted a frictional torque of $0.060 \mathrm{~N} \cdot \mathrm{~m}$ ?
142. [II] A water turbine (having a moment-of-inertia of 1000 $\mathrm{kg} \cdot \mathrm{m}^{2}$ ) is essentially a wheel with a lot of blades attached to it much like its predecessor, the waterwheel. When a high-speed blast of water hits the blades, it drives the turbine into rotation. Suppose the input valve is closed and the turbine winds down from its operating speed of 200 rpm , coming to rest in 30 minutes. Determine the frictional torque acting.
143. [II] A hoop of mass $m$ and radius $R$ rolls without slipping down an incline at an angle of $\theta$. Write an expression for the acceleration of its $c . m$. and find its numerical value when $m=1.0 \mathrm{~kg}, R=1.0 \mathrm{~m}$, and $\theta=30^{\circ}$.
144. [III] Figure P144 shows two masses strung over a $0.50-\mathrm{m}$-diameter pulley whose moment-of-inertia is $0.035 \mathrm{~kg} \cdot \mathrm{~m}^{2}$. If there is a constant frictional torque of $0.25 \mathrm{~N} \cdot \mathrm{~m}$ at the bearing, compute the acceleration of either mass. (The tension in the rope is not the same on either side of the pulley.)
145. [III] Consider a long slender rod of mass $M$ and length $l$ pivoted at its far end at


Figure P144
point $O$. Now compute $I_{0}$ directly by dividing the rod into 100 equalmass parts and summing up the moments of inertia for all of these. You will need to know that

$$
1^{2}+3^{2}+5^{2}+\cdots+(2 n-1)^{2}=\frac{n}{3}(2 n+1)(2 n-1)
$$

146. [III] A solid cylindrical pulley of mass $m$ and radius $R$ is held upright, and a number of circular turns of light cord are wrapped around it within the groove. The loose end of the cord is held vertically, and the pulley is released so it falls straight downward, unwinding like a yo-yo. Please compute its linear acceleration, in terms of $g$, and the tension in the cord, in terms of $m$ and $g$, before it reaches the end of its rope.
147. [III] A solid cylinder is placed at rest on an inclined plane at a vertical height $h$ and allowed to roll down without slipping. Using forces and torques show that its speed at the bottom is given by $v=2 \sqrt{g h / 3}$. Look at Problem 156.
148. [III] A cylinder of radius $R$ and mass $m$ mounted with an axle is placed on a horizontal surface. It's pulled forward via a mechanical arrangement so that the applied horizontal force $F$ passes through the c.m. Assuming the cylinder rolls without slipping, determine both its acceleration and the friction force at the surface in terms of $F$.

## SECTION 8.9: ROTATIONAL KINETIC ENERGY

149. [I] Determine the kinetic energy of a nontranslating disk that is spinning around its central symmetry axis at 300 rpm . The disk has a moment-of-inertia of $1.00 \mathrm{~kg} \cdot \mathrm{~m}^{2}$.
150. [I] A spherical space satellite having a moment-of-inertia of $250 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ is to be spun up from rest to a speed of 12 rpm . How much energy must be imparted to the satellite?
151. [I] Calculate the angular speed of a uniform solid disk about its central symmetry axis if its rotational kinetic energy is 100 J and its moment-of-inertia is $10.0 \mathrm{~kg} \cdot \mathrm{~m}^{2}$.
152. [I] A small lead sphere of mass 0.250 kg is whirling in a horizontal circle at the end of a $2.00-\mathrm{m}$ long string at a speed of 10.0 $\mathrm{rad} / \mathrm{s}$. Approximately what is the value of its rotational KE? If the string breaks, what will be its linear KE immediately thereafter? 153. [I] If the dimensions of mass, length, and time are $M, L$, and $T$, what are the dimensions of angular kinetic energy?
153. [I] A solid rubber ball of radius $R$ and mass $m$ is thrown with a speed $v$. If it leaves the pitcher's hand spinning at a rate $\omega$, write an expression for its total KE.
154. [II] A hollow cylinder, or hoop, of mass $m$ rolls down an inclined plane from a height $h$. If it begins at rest, show that its final speed is given by

$$
v=\sqrt{g h}
$$

156. [II] A solid cylinder of mass 2.0 kg rolls without slipping down a long curved track from a height of 10.0 m . Calculate the linear speed with which it exits the track at the bottom. Look at Problem 147.
157. [II] What is the total kinetic energy of a solid ball of mass $m$ rolling along a horizontal surface at a constant speed $V$ ?
158. [II] With Problem 157 in mind, what fraction of the total energy of a solid sphere rolling at a constant speed exists as translational KE?
159. [II] A long pencil is balanced straight up on its point on a horizontal surface. Without slipping, the pencil topples over. Show that
the speed at which the eraser end strikes the surface is

$$
v=\sqrt{3 g L}
$$

[Hint: Take the pencil to be a uniform rod and apply Conservation of Energy to the c.g.]

## SECTION 8.10: ANGULAR MOMENTUM <br> SECTION 8.11: CONSERVATION OF ANGULAR MOMENTUM

160. [I] A uniform disk of mass 800 kg and radius 0.50 m is rotating around its central symmetry axis at a rate of 60 rpm . Determine its angular momentum.
161. [I] A tiny sphere of mass 50.0 g is a vertical distance of 1.00 m from a point $P$. If at that instant the particle is moving horizontally at a speed of $2.50 \mathrm{~m} / \mathrm{s}$, what if any is its angular momentum with respect to $P$ ?
162. [I] Determine the angular momentum of a hoop having a mass of 2.00 kg and a diameter of 2.00 m rolling down an incline at 2.50 $\mathrm{m} / \mathrm{s}$.
163. [I] A gymnast in the process of performing a forward somersault in midair increases his angular velocity by $450 \%$ while going from an arms-overhead layout position to a tight tuck. What can you say about the change, if any, in the moment-of-inertia about the frontal axis (parallel to the shoulder-to-shoulder line) passing through his c.m. (in one side and out the other)?
164. [I] An astronaut of mass 70 kg rotates his arms backward about a shoulder-to-shoulder axis at a rate of $1 \mathrm{rev} / \mathrm{s}$. Both of his arms together have a mass of $12.5 \%$ of his body mass and a total moment-of-inertia about the shoulders of $1 \mathrm{~kg} \cdot \mathrm{~m}^{2}$. Compute the angular momentum imparted to the rest of his body in the process.
165. [I] A tiny hummingbird with a mass of only $2 \times 10^{-3} \mathrm{~kg}$ is circling around a flower in a $1.0-\mathrm{m}$-diameter orbit. If it travels once around in 1.0 s , approximate its angular momentum with respect to the blossom (look at Problem 132).
166. [II] Compute the orbital angular momentum of Jupiter ( $M_{2}=$ $1.9 \times 10^{27} \mathrm{~kg}, r_{21}=7.8 \times 10^{11} \mathrm{~m}$, and $\left.v_{\mathrm{av}}=13.1 \times 10^{3} \mathrm{~m} / \mathrm{s}\right)$ and then compare it to the spin angular momentum of the Sun $\left(M_{\odot}=1.99 \times 10^{30} \mathrm{~kg}, R_{\odot}=6.96 \times 10^{8} \mathrm{~m}\right.$ ). Assume the Sun, whose equator rotates once in about 26 days, is a rigid sphere of uniform density. It would seem that most of the angular momentum of the Solar System is out there with the giant planets that also rotate quite rapidly.
167. [II] It is possible for a large star (one greater than 1.4 solar masses) to gravitationally collapse, crushing itself into a tiny neutron star perhaps 40 km in diameter. Suppose such a thing could happen to the Sun, which is $1.39 \times 10^{9} \mathrm{~m}$ in diameter and spinning at a rate of about once around every 27 days. What would be the spin rate for the resulting neutron star?
168. [II] A small mass $m$ is tied to a string and swung in a horizontal plane. The string winds around a vertical rod as the mass revolves, like a length of jewelry chain wrapping around an outstretched finger. Given that the initial speed and length are $v_{\mathrm{i}}$ and $r_{\mathrm{i}}$, compute $v_{\mathrm{f}}$ when $r_{\mathrm{f}}=r_{\mathrm{i}} / 10$.
169. [II] A very thin $1.0-\mathrm{kg}$ disk with a diameter of 80 cm is mounted horizontally to rotate freely about a central vertical axis. On the edge of the disk, sticking out a little, is a small, essentially massless, tab or "catcher." A $1.0-\mathrm{g}$ wad of clay is fired at a speed of $10.0 \mathrm{~m} / \mathrm{s}$ directly at the tab perpendicular to it and tangent to the disk. The clay sticks to the tab, which is initially at rest, at a distance of 40 cm from the axis. What is the moment-of-inertia (a) of the clay about the axis? (b) of the disk about the axis? (c) of both clay and disk about the axis? (d) What is the linear momentum of the clay before impact? (e) What is the angular momentum of the clay with respect to the axis just before impact? (f) What is the angular speed of the disk after impact?
170. [II] Write an expression for the orbital angular momentum of a small artificial satellite of mass $m$ in a circular flight-path of radius $r$ about the Earth. Check the units. What happens to angular momentum as the orbit increases?
171. [III] An astronaut is working out at the far reach of a $100-\mathrm{m}$ tether, the opposite end of which is attached to a pivoting ring on the nose of the space station. She does not notice that an air hose on her backpack has developed a leak. The little blast of gas produces a tangential thrust and a corresponding acceleration of $a_{\mathrm{T}}=1.0 \times 10^{-3} \mathrm{~g}$. After two minutes, she realizes what has happened and shuts off the leak. At that point, she and her life support system (total mass of 150 kg ) are sailing along at a tangential speed of __ with an angular momentum of —_ Annoyed, she decides to return to the craft by pulling hand-over-hand on the tether. If she manages to get 5.0 m from the ring on the ship, her tangential speed would be Considering that the centripetal force she would have to hold against is then $\qquad$ , it's unlikely she would ever get that close.

[^0]:    SOLUTION: (a) The sum of the horizontal forces must be zero, which means that the horizontal components of the tensile forces must cancel each other, leaving only their vertical components. These must combine to form a resultant that is equal in magnitude and opposite in direction to the weight, since the sum of the vertical forces must also be zero. (b) The entire system (ropes and body) experiences two forces: a force up (along the plumb line) exerted by the ring, and the weight of the system down. If these were not colinear, the body would experience a nonzero torque and rotate.

